$$\beta^m_{k+1}\, p^m_{k+1}(t) = (t-\alpha^m_k)\, p^m_k(t) - \beta^m_k\, p^m_{k-1}(t) \qquad k\in\mathbb{N}_0,\ t\in\Gamma_m := [-1,1]$$

(1) $\int_{\Gamma_m} \beta_{k+1}\, p_{k+1}(t)\, p_{k+1}(t)\, d\mu_{\Gamma_m} = \int_{\Gamma_m} t\, p_k\, p_{k+1} - \alpha_k \underbrace{\int p_k\, p_{k+1}}_{=0} - \beta_k \underbrace{\int p_{k-1}\, p_{k+1}\, dt}_{=0}$

$$\beta_{k+1} = (t\,p_k, p_{k+1})_{\pi_m} \equiv (t\,p_{k+1}, p_k)_{\pi_m}$$

(2) $\int_{\Gamma_m} \underbrace{\beta_{k+1}\, p_{k+1}\, p_k}_{=0} = \int_{\Gamma_m} t\, p_k\, p_k - \alpha_k \underbrace{\int_{\Gamma_m} p_k\, p_k}_{=1} - \beta_k \underbrace{\int_{\Gamma_m} p_{k-1}\, p_k}_{=0} \Rightarrow \boxed{\alpha_k = (t\,p_k, p_k)_{\pi_m}}$

(3) $\int_{\Gamma_m} \underbrace{\beta_{k+1}\, p_{k+1}\, p_{k-1}}_{=0} = \int_{\Gamma_m} t\, p_k\, p_{k-1} - \alpha_k \underbrace{\int_{\Gamma_m} p_k\, p_{k-1}}_{=0} - \beta_k \underbrace{\int_{\Gamma_m} p_{k-1}\, p_{k-1}}_{=1} \Rightarrow \boxed{\beta_k = (t\,p_k, p_{k-1})_{\pi_m}}$

$$\alpha_k = \int_{\Gamma_m} t\, p_k(t)\, p_k(t)\, d\mu_{\Gamma_m} \equiv \int_{-1}^{1} t\, p_k(t)\, p_k(t)\, \rho_m(t)\, dt \qquad (*)$$

Therefore, with ρ_m even:

$f(t) = t\, p^e_k(t)\, \rho_m(t)$

$\Downarrow$

$f(-t) = (-t)\, \underbrace{p^2_k(-t)}_{=p^2_k(t)}\, \underbrace{\rho_m(-t)}_{=\rho_m(t)}$

$= -t\, \underbrace{p^2_k(t)\, \rho_m(t)}_{=f(t)} \equiv -f(t)$

$\Downarrow$

$f(t) \equiv -f(-t)$ i.e. is odd

Thus $\alpha_k \equiv 0$

If $f(t)$ is even is even if $f(t) \equiv f(-t)$	$f(t)$ is even if $f(t) = -f(-t)$

Then:

$p^2_k(t) = p_k(t)\, p_k(t)$

if p_k is odd $\Rightarrow$ $p^2_k(-t) = \underbrace{p_k(-t)}_{=-p_k(t)}\, \underbrace{p_k(-t)}_{=-p_k(t)} \equiv +p^2_k(t)$

if p_k is even $\Rightarrow$ $p^2_k(-t) = \underbrace{p_k(-t)}_{=p_k(t)}\, \underbrace{p_k(-t)}_{=p_k(t)} \equiv +p^2_k(t)$

$\Downarrow$ In both cases $p^2_k(-t) \equiv p^2_k(t)$

i.e., $p^2_k(t)$ is even

Ergo 3-term rec. becomes

$$\beta^m_{k+1}\, p^m_{k+1}(t) = t\, p^m_k(t) - \beta^m_k\, p^m_{k-1}(t)$$